Engineering Econ Review

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Engineering Econ allows us to compare money spent or received (cash flows) in the future to an amount of money spent today. The most familiar example is an automobile loan. Borrow $20,000 today to buy a car and pay it off in a series of 60 equal payments. How do they decide on the size of my payment? Engineering econ. As engineers we are constantly faced with decisions based on an investment today vs. a benefit in the future.

Basics:

I or i = the interest rate per interest period
i_n = the nominal interest rate, often called r or APR (annual % rate)
i_e = effective interest rate
n = number of compounding periods (often years, but not always)
m = number of compounding periods in one year
EOY = end of year (BOY is beginning of year)
Payback period is the amount of time required to recoup your initial investment.
Return on investment is the interest rate that makes 2 investments equivalent
PV = present value   FV = future value
Annuity (A) is a series of equal payments
Gradient (G) is an increasing series, either arithmetic or geometric
In the tables: n is the number of periods
P is present value, F is future value, A is annuity, G is gradient
The P/F column is used to find (P)resent value given a (F)uture value
The P/A column is used to find (P)resent value given an (A)nnuity, etc.

Keys to success in solving most engineering econ problems:

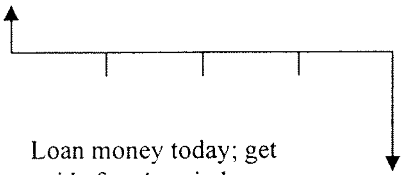
- Draw a cash flow diagram
- Identify (P)resent values, (F)uture values, (A)nnuities, (i)nterest rate and (n)umber of periods (not all will be given)
- Calculate the present value of each amount, or the annuity amount, or the future amount
- Compare results

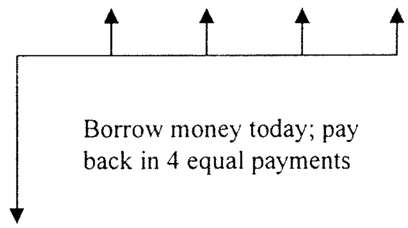
Resource: http://www.feexam.ou.edu/ there are reviews of every area on the FE. I only used a few of the example questions here.

Effective interest rate vs. nominal. $i_e = (1+ i_n/m)^m$ For example 9% nominal annual interest compounded monthly is $(1+0.09/12)^{12}-1 = 0.0938 = 9.38\%$ effective.

Another type is: X% annual growth with payments every Y years is an effective rate of $(1+X)^Y-1$. If costs grow by 3.5% every year, and we re-pave a road every 5 years, then the re-paving cost increases by $(1+0.035)^5-1 = 18.77\%$ every 5 years.

Cash flow diagram = a drawing that is used to help understand the size and timing of expenses and revenues. Be sure you are consistent; i.e. all cash flows in to you should be in the same direction.





Single payment now vs. single payment in the future. We expect our money to grow if we lend it, so a payment in the future should be larger than a payment today. It grows by $(1+i_e)^n$. On the other hand, if we know we will receive money in the future, we would accept *less* at the present instead. The reduction is the reciprocal of the last equation, $1/(1+i_e)^n$.

1.217

Borrow \$100 today at 4% interest. How much do you owe after 5 years? *(\$121.67)*

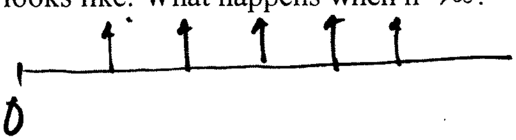
$\$100 \times (1.04)^5 = 121^{\underline{67}}$ $\$100\,(F/P, 4\%, 5\text{yrs})$

$\doteq \$121^{\underline{70}}$

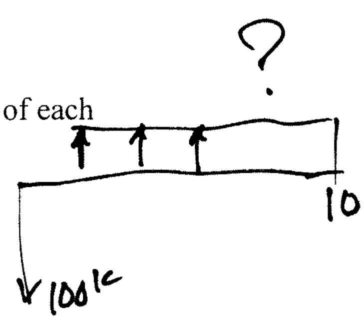
You win \$1,000, payable one year from now. How much would you take today, instead, if interest is 6%? *(\$943.40)*.

$\$1000/(1.06)^1 = 943^{\underline{40}}$

One present payment vs. a uniform series of payments. Be sure you understand what the cash flow diagram looks like. What happens when $n \to \infty$?

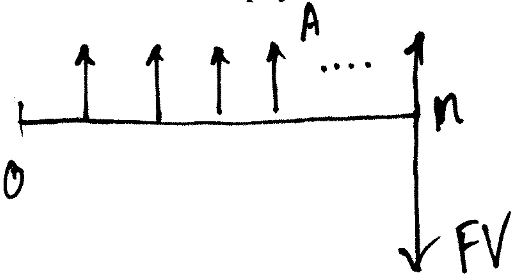


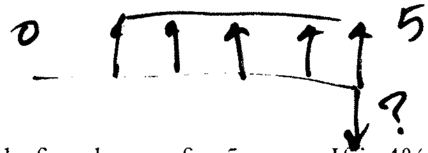
You borrow \$100,000 today, to be paid back in 10 equal payments at the end of each year, at 6% interest. How much are the payments? *(\$13,590)*.



$\$100^k\,(\underbrace{A/P, 6\%, 10\text{yrs}}_{0.1359}) = \$13{,}590^-$

One future payment vs. a uniform series of payments. Cash flow diagram (this one does not look "logical").



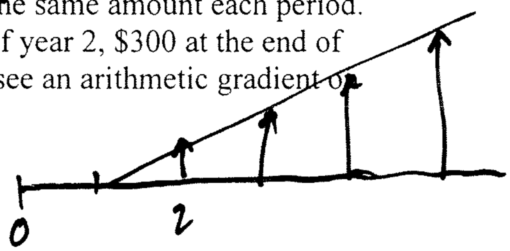


You invest \$10,000 at the end of each year for 5 years. If i=4%, what is the value of the account at EOY 5? *(\$54,163).*

$\$10^k\,(F/A,\ 4\%,\ 5\text{yrs}) = \$54{,}160$

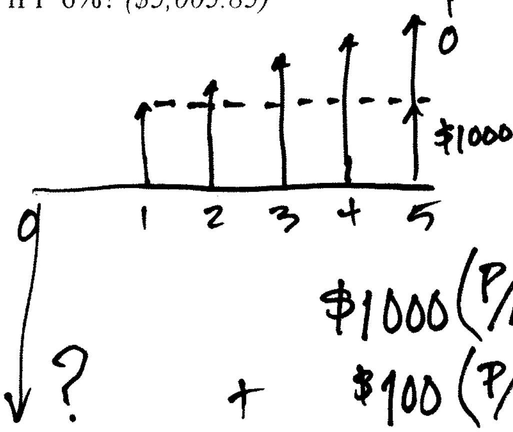
5.416

Arithmetic Gradients. Payments increase or decrease by the same amount each period. For example, \$100 at the end of year one, \$200 at the end of year 2, \$300 at the end of year 3, etc. Cash flow diagram begins at EOY 2. You may see an arithmetic gradient on top of an annuity.



What is the present value if i=6%? *(\$5,005.85)*

EOY	Receipt
1	\$1,000
2	\$1,100
3	\$1,200
4	\$1,300
5	\$1,400



4.2124

$\$1000\,(P/A,\ 6\%,\ 5\text{yrs})$

$+\ \$100\,(P/G,\ 6\%,\ 5\text{yrs}) = \$5{,}005.85$

7.9345

Capitalized Costs, n→∞. When n becomes "very large," assume that it goes to infinity. Future values generally go to infinity, but in many cases present values tend toward limit. Capitalized costs go to A/i.

$PV = \dfrac{A}{i}$ if $n \to \infty$

Maintenance on traffic lights costs \$5,000 per year, and is expected to be required for a very long time. If i=6%, what is the capitalized cost?

$\dfrac{\$5{,}000}{0.06} = \$83{,}333.33$

PTZ≠ETZ. Understand what the cash flow diagrams look like. If the problem cash flow diagram does not look like the cash flow diagrams of any of your equations, you will have to use more than one step to find the answer.

$= \$5858$

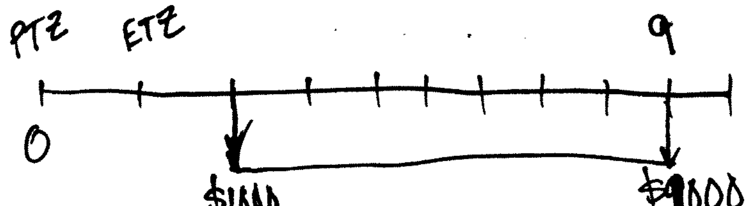
I just purchased a machine. It has a 2 year warranty. I can purchase an extended warranty until the machine is 10 years old by making payments of \$1000 per year, at the beginning of each year starting when the original warranty runs out, or by paying \$7,000 today. Use an interest rate of 6% and determine which option is better.

6.2098

$\$1000\,(P/A,\ 8\text{yrs},\ 6\%)$

$\times\ (P/F,\ 1\text{yr},\ 6\%)$

0.9434



Depreciation: Depreciation is the decrease in value of an asset. Straight line is the simplest form. The depreciation each year is equal to (Cost – Salvage)/life. Accelerated schemes are available, including MACRS. In this case the depreciation is equal to a factor from the table times Cost.

Book Value: BV is the Cost minus accumulated depreciation.

What is the depreciation each year for a trailer that cost $10,000 new, has a life of 5 years, and a salvage value of $2,000? Use straight line depreciation. What is the book value after 2 years? S.L. $\frac{\$10,000-2,000}{5\text{ yrs}} = \$1600/\text{yr}$

BV_1 10000 − 1600 = $8400 BV_2 8400 − 1600 = $6800

MACRS (5YR CLASS) $10,000

	DEP	$	BV
YR 1	$10,000 × 0.2 =	$2,000	10,000 − 2000 = $8k
YR2	$10,000 × 0.32 =	$3200	$8k − $3200 = $4800

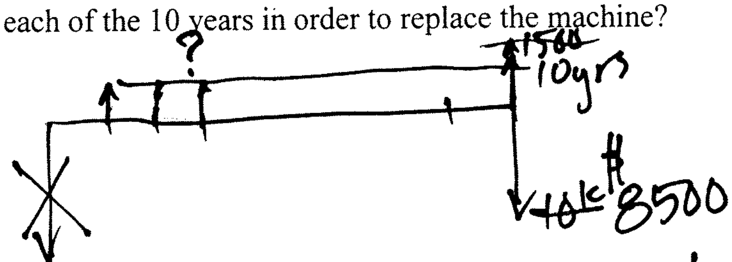
Example Problems

An amount P is invested at interest rate i per compounding period. F is the account balance after n compounding periods. Select the formula that relates F to P.

(A) ~~$F=P(1+i)^{n+1}$~~
(B) $F=P(1+i)^n$ ←
(C) ~~$F=P(1+i)^i$~~
(D) $F=P(1+i)^{-n}$

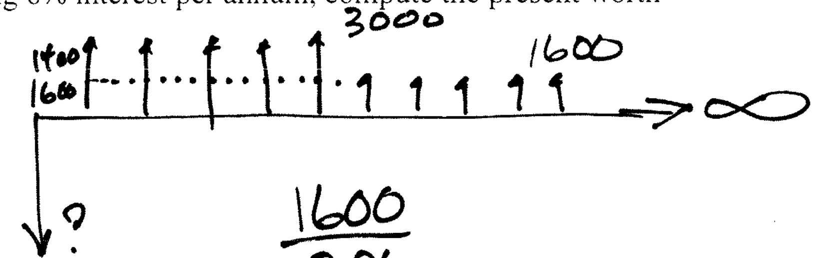
A solar heating system costs $10,000, has an estimated life of 10 years and a scrap value of $1500. Assuming no inflation and an interest rate of 4%, what uniform annual amount must be invested at the end of each of the 10 years in order to replace the machine?

(A) $708
(B) $850
(C) $1000
(D) $1152



$8500(A/F, 4%, 10yrs) = $708
0.0833

An investment has infinite life and makes annual payments of $3000 for the first 5 years and $1600 per year thereafter. Using 6% interest per annum, compute the present worth of the annual disbursements.



(A) $15,000
(B) $25,000
(C) $32,600
(D) $50,200

$$\frac{1600}{0.06}$$

$$+\ 1400(P/A,\ 5\text{yrs},\ 6\%) = \$32564$$

$$4.2124$$

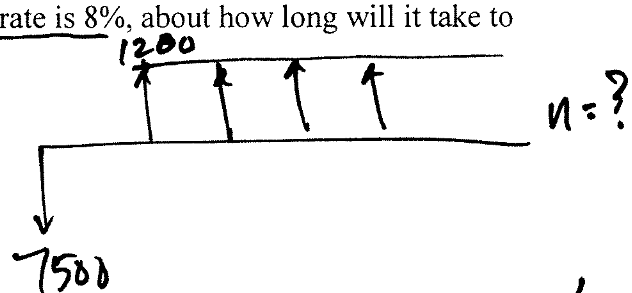
Interest on a debt is 12% per year compounded monthly. Compute the effective annual interest rate.

(A) 1%
(B) 12%
(C) 12.7%
(D) 13.2%

$$\left(1+\frac{0.12}{12\text{mo}}\right)^{12\text{mo}} - 1 = 0.127$$

12.7%

A more efficient heating system adds $7,500 to the cost of your project. Adding this system will save $1200 per year. If the discount rate is 8%, about how long will it take to pay back the initial investment?



(A) 6 years
(B) 7 years
(C) 8 years
(D) 9 years
(E) 10 years

$+\ 1200/1.08^7$

YR 1 $1200/1.08 =$

2 $1200/1.08^2 =$

3 $1200/1.08^3 =$

4 $1200/1.08^4 =$

5 $1200/1.08^5 =$

6 $1200/1.08^6 =$

Σ

Probability: A culvert is required under a highway. If the highway is overtopped, it will cost $20k to repair. If i=6% and the life of each culvert is 20 years, which of the following is most economical?

Type of culvert	Cost new	Probability of overtopping in any one year
24 inch RCP	$20,000	0.20 × $20k = $4000
2 ft by 4 ft box	$40,000	0.10 × $20k = $2000
Twin 2 ft by 4 ft box	$60,000	0.05 × $20k = $1000

$20,000 + $4,000(P/A, 20yrs, 6%) = $65,879

$40k + $2000 (P/A, 20yrs, 6%) = $62,939 ←

$60k + 1000 (P/A, 20yrs, 6%) = $71,470

11.4699

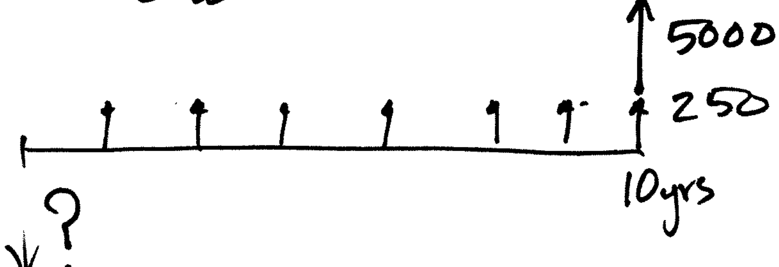
If the sum of $12,000 is borrowed and the debtor is obligated to pay the creditor $900 for each year the loan is in existence, then, the simple interest is: *I would* <u>*have to*</u> *assume that the $12,000 is paid back at the end of the loan period*

$$\frac{\$900}{\$12,000} = 0.075 = 7.5\%$$

A Sprawl Transportation Authority bond has par value of $5000 and term of 10 years. The bond pays 5% nominal annual interest on par value. Estimate the selling price of the bond if the market interest rate is 6%.

(A) $4632
(B) $5000
(C) $7500
(D) $8000

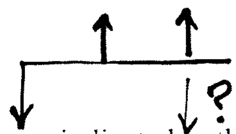
5% × $5000 = $250 ANNUALLY



$250(P/A, 6%, 10yrs) + $5,000(P/F, 6%, 10yrs)

~~6.8017~~ 7.36 0.5584

$4632



Two alternative investments have the cash flows indicated in the table. At 6% interest, which alternative should be selected based on future value? (negative is a cost, positive a benefit)

Year	Alternate M	Alternate N
0	-$1,830	-$2,350
1	$1,000	$1,200
2	$1,000	$1,200

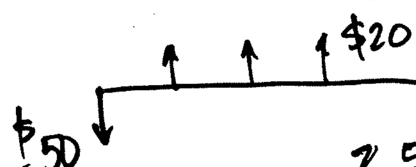
M $\$1000(F/A, 2\text{yrs}, 6\%) - 1830(F/P, 2\text{yrs}, 6\%) = \$4^{91}$

N $\$1200(F/A, 2\text{yrs}, 6\%) - \$2{,}350(F/P, 2\text{yrs}, 6\%) = -\$167^{05}$

MIN ATTRACTIVE RATE OF RETURN

Consider two alternatives, A and B, having cash flows as shown in the figure. If the MARR is 8%, determine which alternative should be selected using the benefit cost analysis method.

Year	A	B	A - B
0	-$200	-$150	-$50
1	$85	$65	$20
2	$85	$65	$20
3	$85	$65	$20



$$\frac{\$20\overbrace{(P/A, 3\text{yrs}, 8\%)}^{2.577}}{\$50} \qquad \frac{51^{54}}{50} = 1.031$$

CHOOSE A

Break even analysis on the FE exam is most likely a simple problem with no discounting (no interest). If there is no interest rate, then it is simply a matter of comparing cost to income, where cost is usually fixed cost + variable cost*units and income is usually income per unit*units.

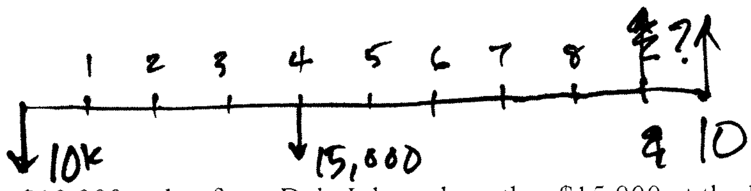
Corp Inc. produces 4,000 robots per year. Their fixed costs are $1,000,000 per year, and the variable cost per robot is $2,500. The selling price is $10,000 per robot. Find the breakeven point and gross profit at this maximum capacity.

$$\$1{,}000{,}000 + \$2{,}500X = \$10{,}000X$$

$$\$1M = 7500x \Rightarrow X = \underline{133.33}$$

GROSS PROFIT

$(\$10^{K} - 2{,}500)4000 - \$1{,}000{,}000 = \$29$ MILLION



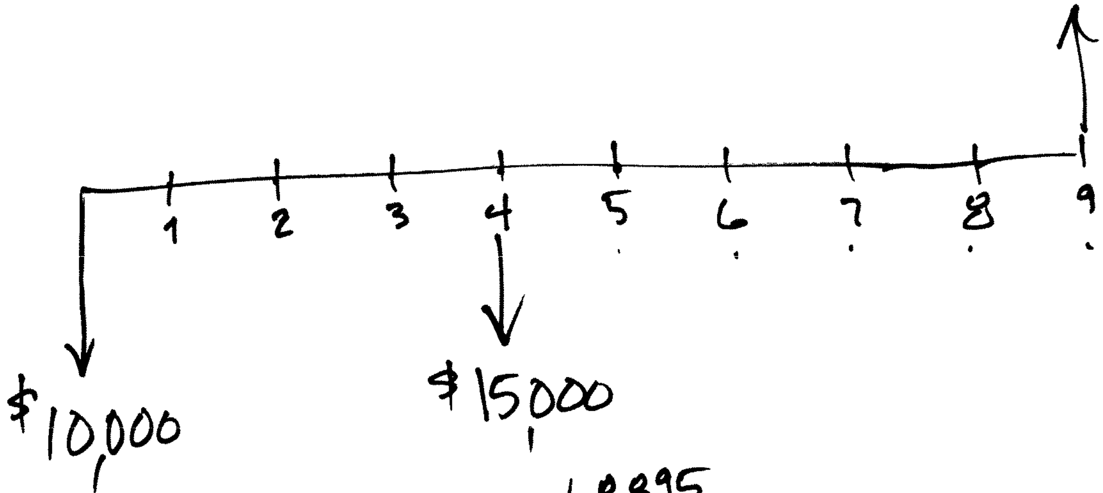
Bill Ding borrows \$10,000 today from Dale Ight and another \$15,000 at the beginning of year 5. The agreed interest rate is 6%, and the loan is to be repaid in full at the end of year 9. What amount will Bill owe Dale to pay off the loan?
~~*End of year 9 is the beginning of year 10.*~~

4% Compound Interest Factors 4%

	SINGLE PAYMENT		UNIFORM PAYMENT SERIES				GRADIENT SERIES	
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth
n	Find F Given P F/P	Find P Given F P/F	Find A Given F A/F	Find A Given P A/P	Find F Given A F/A	Find P Given A P/A	Find A Given G A/G	Find P Given G P/G
1	1.040	.9615	1.0000	1.0400	1.000	.962	0	0
2	1.082	.9246	.4902	.5302	2.040	1.886	.490	.925
3	1.125	.8890	.3203	.3603	3.122	2.775	.974	2.703
4	1.170	.8548	.2355	.2755	4.246	3.630	1.451	5.267
5	1.217	.8219	.1846	.2246	5.416	4.452	1.922	8.555
6	1.265	.7903	.1508	.1908	6.633	5.242	2.386	12.506
7	1.316	.7599	.1266	.1666	7.898	6.002	2.843	17.066
8	1.369	.7307	.1085	.1485	9.214	6.733	3.294	22.181
9	1.423	.7026	.0945	.1345	10.583	7.435	3.739	27.801
10	1.480	.6756	.0833	.1233	12.006	8.111	4.177	33.881
11	1.539	.6496	.0741	.1141	13.486	8.760	4.609	40.377
12	1.601	.6246	.0666	.1066	15.026	9.385	5.034	47.248
13	1.665	.6006	.0601	.1001	16.627	9.986	5.453	54.455
14	1.732	.5775	.0547	.0947	18.292	10.563	5.866	61.962
15	1.801	.5553	.0499	.0899	20.024	11.118	6.272	69.735

6% Compound Interest Factors 6%

	SINGLE PAYMENT		UNIFORM PAYMENT SERIES				GRADIENT SERIES	
	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Compound Amount Factor	Present Worth Factor	Gradient Uniform Series	Gradient Present Worth
n	Find F Given P F/P	Find P Given F P/F	Find A Given F A/F	Find A Given P A/P	Find F Given A F/A	Find P Given A P/A	Find A Given G A/G	Find P Given G P/G
1	1.060	.9434	1.0000	1.0600	1.000	.943	0	0
2	1.124	.8900	.4854	.5454	2.060	1.833	.485	.890
3	1.191	.8396	.3141	.3741	3.184	2.673	.961	2.569
4	1.262	.7921	.2286	.2886	4.375	3.465	1.427	4.946
5	1.338	.7473	.1774	.2374	5.637	4.212	1.884	7.935
6	1.419	.7050	.1434	.2034	6.975	4.917	2.330	11.459
7	1.504	.6651	.1191	.1791	8.394	5.582	2.768	15.450
8	1.594	.6274	.1010	.1610	9.897	6.210	3.195	19.842
9	1.689	.5919	.0870	.1470	11.491	6.802	3.613	24.577
→ 10	1.791	.5584	.0759	.1359	13.181	7.360	4.022	29.602
11	1.898	.5268	.0668	.1268	14.972	7.887	4.421	34.870
12	2.012	.4970	.0593	.1193	16.870	8.384	4.811	40.337
13	2.133	.4688	.0530	.1130	18.882	8.853	5.192	45.963
14	2.261	.4423	.0476	.1076	21.015	9.295	5.564	51.713
15	2.397	.4173	.0430	.1030	23.276	9.712	5.926	57.555



1.6895

$$\$10{,}000\,(1+0.06)^9 = 16{,}895$$

$$+\quad \$15{,}000\,(1.06)^5 \qquad 20{,}073$$

1.3382

$$\$36{,}968$$

ENGINEERING ECONOMICS

Factor Name	Converts	Symbol	Formula
Single Payment Compound Amount	to F given P	$(F/P, i\%, n)$	$(1 + i)^n$
Single Payment Present Worth	to P given F	$(P/F, i\%, n)$	$(1 + i)^{-n}$
Uniform Series Sinking Fund	to A given F	$(A/F, i\%, n)$	$\dfrac{i}{(1 + i)^n - 1}$
Capital Recovery	to A given P	$(A/P, i\%, n)$	$\dfrac{i(1 + i)^n}{(1 + i)^n - 1}$
Uniform Series Compound Amount	to F given A	$(F/A, i\%, n)$	$\dfrac{(1 + i)^n - 1}{i}$
Uniform Series Present Worth	to P given A	$(P/A, i\%, n)$	$\dfrac{(1 + i)^n - 1}{i(1 + i)^n}$
Uniform Gradient Present Worth	to P given G	$(P/G, i\%, n)$	$\dfrac{(1 + i)^n - 1}{i^2(1 + i)^n} - \dfrac{n}{i(1 + i)^n}$
Uniform Gradient † Future Worth	to F given G	$(F/G, i\%, n)$	$\dfrac{(1 + i)^n - 1}{i^2} - \dfrac{n}{i}$
Uniform Gradient Uniform Series	to A given G	$(A/G, i\%, n)$	$\dfrac{1}{i} - \dfrac{n}{(1 + i)^n - 1}$

NOMENCLATURE AND DEFINITIONS

A.......... Uniform amount per interest period
B.......... Benefit
BV........ Book value
C.......... Cost
d........... Combined interest rate per interest period
D_j......... Depreciation in year j
F.......... Future worth, value, or amount
f........... General inflation rate per interest period
G.......... Uniform gradient amount per interest period
i .... Interest rate per interest period
i_e .... Annual effective interest rate
m.......... Number of compounding periods per year
n........... Number of compounding periods; or the expected life of an asset
P.......... Present worth, value, or amount
r........... Nominal annual interest rate
S_n .... Expected salvage value in year n

Subscripts

j .... at time j
n........... at time n
†........... $F/G = (F/A - n)/i = (F/A) \times (A/G)$

NON-ANNUAL COMPOUNDING

$$i_e = \left(1 + \frac{r}{m}\right)^m - 1$$

BREAK-EVEN ANALYSIS

By altering the value of any one of the variables in a situation, holding all of the other values constant, it is possible to find a value for that variable that makes the two alternatives equally economical. This value is the break-even point.

Break-even analysis is used to describe the percentage of capacity of operation for a manufacturing plant at which income will just cover expenses.

The payback period is the period of time required for the profit or other benefits of an investment to equal the cost of the investment.

INFLATION

To account for inflation, the dollars are deflated by the general inflation rate per interest period f, and then they are shifted over the time scale using the interest rate per interest period i. Use a combined interest rate per interest period d for computing present worth values P and Net P.
The formula for d is $d = i + f + (i \times f)$